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Comment on "Dispersion relation for MHD waves in homogeneous plasma"

Suresh Chandra, G.M. Dak, B.K. Kumthekar and Monika Sharma

School of Physics, Shri Mata Vaishno Devi University, Katra 182 320 (J&K), India

Email: suresh492000@yahoo.co.in

Abstract: Pandey & Dwivedi (2007) again tried to claim that the dispersion relation for the given set of equations must be a sixth degree polynomial. Through a series of papers, they are unnecessarily creating confusion. In the present communication, we have shown how Pandey & Dwivedi (2007) are introducing an additional root, which is insignificant. Moreover, five roots of both the polynomials are common and they are sufficient for the discussion of propagation of slow-mode and fast-mode waves.

Keywords: MHD - Sun: Corona - waves

1 Introduction

For application of magnetohydrodynamics (MHD) in solar physics as well as in plasma physics, dispersion relation plays key role. The basic equations under the present investigation can be expressed as (Pandey & Dwivedi, 2007, hereinafter referred to as PD)

$$\frac{\partial \rho}{\partial t} + \nabla . \, (\rho \vec{v}) = 0 \tag{1}$$

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \frac{1}{4\pi}(\nabla \times \vec{B}) \times \vec{B} - \nabla . \Pi \tag{2}$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) \tag{3}$$

$$\frac{Dp}{Dt} + \gamma p (\nabla . \, \vec{v}) = (\gamma - 1)\Big[\nabla . \, \kappa \nabla T + Q_{vis} - Q_{rad}\Big] \tag{4}$$

$$p = \frac{2\rho k_B T}{m_p} \tag{5}$$

Here, symbols have their usual meaning. The quantities Q_{th}, Q_{vis} and Q_{rad} are

$$Q_{th} = \kappa_{\|}\Big(\frac{\partial T}{\partial z}\Big)^2 T^{-1} \qquad Q_{vis} = \frac{\eta_0}{3}(\nabla . \, \vec{v})^2 \qquad Q_{rad} = n_e n_H Q(T)$$

where $\kappa_{\|}$ represents the conductivity along the magnetic field and is expressed by $\kappa_{\|} \approx 10^{-6} T^{5/2}$. For small perturbations from the equilibrium, we have

$$\rho = \rho_0 + \rho_1 \qquad \vec{v} = \vec{v}_1 \qquad \vec{B} = \vec{B}_0 + \vec{B}_1$$
$$p = p_0 + p_1 \qquad T = T_0 + T_1 \qquad \Pi = \Pi_0 + \Pi_1$$

where the equilibrium part is denoted by the subscript '0' and the perturbation part by the subscript '1'. For the magnetic field taken along the z-axis, ($i.e.$, $\vec{B}_0 = B_0 \hat{z}$)

and the propagation vector $\vec{k} = k_x\hat{x} + k_z\hat{z}$, the equations (1) - (5) can be linearized in the following form (Chandra and Kumthekar, 2007):

$$\frac{\partial \rho_1}{\partial t} + \rho_0(\nabla . \vec{v}_1) = 0 \tag{6}$$

$$\rho_0 \frac{\partial \vec{v}_1}{\partial t} = -\nabla p_1 + \frac{1}{4\pi}(\nabla \times \vec{B}_1) \times \vec{B}_0 - \nabla . \Pi_0 \tag{7}$$

$$\frac{\partial \vec{B}_1}{\partial t} = \nabla \times (\vec{v}_1 \times \vec{B}_0) \tag{8}$$

$$\frac{\partial p_1}{\partial t} + \gamma p_0(\nabla . \vec{v}_1) + (\gamma - 1)\kappa_{\parallel} k_z^2 T_1 = 0 \tag{9}$$

$$\frac{p_1}{p_0} = \frac{\rho_1}{\rho_0} + \frac{T_1}{T_0} \tag{10}$$

For the perturbations that are proportional to $\exp[i(\vec{k} . \vec{r} - \omega t)]$, equations (6) - (10) reduce to the following equations:

$$\omega\rho_1 - \rho_0(k_x v_{1x} + k_z v_{1z}) = 0 \tag{11}$$

$$\omega\rho_0 v_{1x} - k_x p_1 - \frac{B_0}{4\pi}(k_x B_{1z} - k_z B_{1x}) + \frac{i\eta_0}{3}(k_x^2 v_{1x} - 2k_x k_z v_{1z}) = 0 \tag{12}$$

$$\omega\rho_0 v_{1y} + \frac{B_0}{4\pi}(k_z B_{1y}) = 0 \tag{13}$$

$$\omega\rho_0 v_{1z} - k_z p_1 + \frac{i\eta_0}{3}(4k_z^2 v_{1z} - 2k_x k_z v_{1x}) = 0 \tag{14}$$

$$\omega B_{1x} + k_z B_0 v_{1x} = 0 \tag{15}$$

$$\omega B_{1y} + k_z B_0 v_{1y} = 0 \tag{16}$$

$$\omega B_{1z} - k_x B_0 v_{1x} = 0 \tag{17}$$

$$i\omega p_1 - i\rho_0 c_s^2(k_x v_{1x} + k_z v_{1z}) - (\gamma - 1)\kappa_{\parallel} k_z^2 T_1 = 0 \tag{18}$$

$$\frac{p_1}{p_0} - \frac{\rho_1}{\rho_0} - \frac{T_1}{T_0} = 0 \tag{19}$$

These equations (11) - (19) are the same as the equations (11) - (19) of PD. (In equation (11) of PD, ρ must be ρ_1.) Equations (13) and (16) for the variables v_{1y} and B_{1y} are decoupled from the rest and describe the Alfvén waves. The rest of the equations for p_1, ρ_1, T_1, B_{1x}, B_{1z}, v_{1x} and v_{1z} describe damped magnetohydrostatic waves. Now, on substituting B_{1x} and B_{1z} from equations (15) and (17) in equations (12) and (14), respectively, we get

$$\left(\omega^2\rho_0 + \frac{i\omega\eta_0}{3}k_x^2 - v_A^2\rho_0 k^2\right)v_{1x} - \frac{2i\omega\eta_0 k_x k_z}{3}\, v_{1z} - k_x\omega p_1 = 0 \tag{20}$$

and

$$\frac{2i\eta_0 k_x k_z}{3}\, v_{1x} - \left(\omega\rho_0 + \frac{4i\eta_0}{3}k_z^2\right)v_{1z} + k_z p_1 = 0 \tag{21}$$

When we eliminate ρ_1 and T_1 from equations (11), (18) and (19), we get

$$(c_0 p_0 k_x - i\rho_0 c_s^2 k_x \omega)v_{1x} + (c_0 p_0 k_z - i\rho_0 c_s^2 k_z \omega)v_{1z} - (c_0\omega - i\omega^2)p_1 = 0 \tag{22}$$

where $c_0 = (\gamma - 1)\kappa_{\parallel} k_z^2 T_0/p_0$; $c_s^2 = \gamma p_0/\rho_0$; and $v_A^2 = B_0^2/4\pi\rho_0$. Thus, the equations (20), (21) and (22) are obtained from the equations which are the same as of PD.

2 Dispersion relation

For convenience, let us express equations (20) - (22) as

$$a_{11}v_{1x} + a_{12}v_{1z} + a_{13}p_1 = 0 \tag{23}$$

$$a_{21}v_{1x} + a_{22}v_{1z} + a_{23}p_1 = 0 \tag{24}$$

$$a_{31}v_{1x} + a_{32}v_{1z} + a_{33}p_1 = 0 \tag{25}$$

where the coefficients a's are:

$$a_{11} = \left(\omega^2\rho_0 + \frac{i\omega\eta_0}{3}k_x^2 - v_A^2\rho_0 k^2\right); \qquad a_{12} = -\frac{2i\omega\eta_0}{3}k_x k_z; \qquad a_{13} = -k_x\omega$$

$$a_{21} = \frac{2i\eta_0}{3}k_x k_z; \qquad a_{22} = -\omega\rho_0 - \frac{4i\eta_0}{3}k_z^2; \qquad a_{23} = k_z$$

$$a_{31} = c_0 p_0 k_x - i\rho_0 c_s^2 k_x\omega; \qquad a_{32} = c_0 p_0 k_z - i\rho_0 c_s^2 k_z\omega; \qquad a_{33} = i\omega^2 - c_0\omega$$

For a non-trivial solution of the set of equations (23), (24) and (25), we must have

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = 0$$

Expansion of this determinant and substitution of the values of a's gives the fifth degree polynomial:

$$\omega^5 + iA\omega^4 - B\omega^3 - iC\omega^2 + D\omega + iE = 0 \tag{26}$$

where

$$A = c_0 + \frac{\eta_0}{3\rho_0}(k_x^2 + 4k_z^2)$$

$$B = \frac{c_0\eta_0}{3\rho_0}(k_x^2 + 4k_z^2) + (c_s^2 + v_A^2)k^2$$

$$C = \frac{3\eta_0}{\rho_0}c_s^2 k_x^2 k_z^2 + \frac{c_0 p_0 k^2}{\rho_0} + v_A^2 c_0 k^2 + \frac{4\eta_0 v_A^2 k_z^2 k^2}{3\rho_0}$$

$$D = \frac{3c_0 p_0 \eta_0 k_x^2 k_z^2}{\rho_0^2} + \frac{4\eta_0 c_0 v_A^2 k_z^2 k^2}{3\rho_0} + v_A^2 c_s^2 k_z^2 k^2$$

$$E = v_A^2 c_0 p_0 k_z^2 k^2/\rho_0$$

It obviously shows that the dispersion relation is a fifth degree polynomial. Now, question arises how PD are getting the sixth degree polynomial. It can be understood in the following manner.

3 How PD is getting sixth degree polynomial

The dispersion relation of PD can be derived in the following manner: On eliminating p_1 from equations (23) and (25), we get

$$(a_{11}a_{33} - a_{31}a_{13})v_{1x} + (a_{12}a_{33} - a_{32}a_{13})v_{1z} = 0 \tag{27}$$

On eliminating p_1 from equations (24) and (25), we get

$$(a_{21}a_{33} - a_{31}a_{23})v_{1x} + (a_{22}a_{33} - a_{32}a_{23})v_{1z} = 0 \tag{28}$$

From equations (27) and (28), we have

$$(a_{11}a_{33} - a_{31}a_{13})(a_{22}a_{33} - a_{32}a_{23}) = (a_{21}a_{33} - a_{31}a_{23})(a_{12}a_{33} - a_{32}a_{13}) \tag{29}$$

Substitution of the values of a's in equation (29), gives the dispersion relation

$$\omega^6 + iA'\omega^5 - B'\omega^4 - iC'\omega^3 + D'\omega^2 + iE'\omega - F' = 0 \tag{30}$$

where

$$A' = 2c_0 + c_1; \qquad B' = (c_s^2 + v_A^2)k^2 + c_0(2c_1 + c_0);$$

$$C' = c_2 + c_0(k^2(c_s^2 + 2v_A^2 + \frac{p_0}{\rho_0}) + c_0c_1) \qquad D' = c_s^2c_6 + c_0(c_3 + c_0c_4);$$

$$E' = c_0\Big[c_0c_5 + c_6(c_s^2 + \frac{p_0}{\rho_0})\Big]; \qquad F' = c_0^2c_6p_0/\rho_0$$

and

$$c_1 = \eta_0(k_x^2 + 4k_z^2)/3\rho_0; \qquad c_2 = \eta_0k_z^2(4v_A^2k^2 + 9c_s^2k_x^2)/3\rho_0$$

$$c_3 = \frac{\eta_0k_z^2}{3\rho_0}\Big(8v_A^2k^2 + 9\Big(c_s^2 + \frac{p_0}{\rho_0}\Big)k_x^2\Big) \qquad c_4 = \Big(v_A^2 + \frac{p_0}{\rho_0}\Big)k^2$$

$$c_5 = \frac{\eta_0k_z^2}{3\rho_0}\Big(4v_A^2k^2 + \frac{9p_0k_x^2}{\rho_0}\Big) \qquad c_6 = v_A^2k^2k_z^2$$

This is the dispersion relation derived by PD. Now, here we can show that DP has introduced an addition root in this dispersion relation.

4 Discussion

It can be easily found that

$$\omega^6 + iA'\omega^5 - B'\omega^4 - iC'\omega^3 + D'\omega^2 + iE'\omega - F' = (\omega + ic_0) \times (\omega^5 + iA\omega^4 - B\omega^3 - iC\omega^2 + D\omega + iE)$$

showing that five roots of both equations (26) and (30) are common. The additional root $\omega = -ic_0$ is introduced by PD. The five common roots, are of the form $i\alpha_1$, $-\beta_2 + i\alpha_2$, $\beta_2 + i\alpha_2$, $-\beta_3 + i\alpha_3$ and $\beta_3 + i\alpha_3$. The first root corresponds to the thermal motion whereas the rest four give the slow-mode and fast-mode waves. The sixth root $-ic_0$ also corresponds to the thermal motion. This has no significance, as we are interested only in the slow-mode and fast-mode waves. It is difficult to understand how science is affected when one considers the sixth degree polynomial derived by PD.

It can finally be concluded that the dispersion relation is of fifth degree polynomial. PD got the sixth degree polynomial, as they have introduced an additional root. Moreover, only five roots are sufficient for propagation of slow-mode and fast-mode waves.

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